V. "On the Steam Calorimeter." By J. JOLY, M.A. Communicated by G. F. FITZGERALD, F.R.S., F.T.C.D. Received November 26, 1889.

(Abstract.)

The theory of the method of condensation has been previously given by the author in the 'Proceedings of the Royal Society,' vol. 41, p. 352.

Since the publication of that paper a much more extended knowledge of the capabilities of the method has been acquired, which has led to the construction of new forms of the apparatus, simple in construction and easily applied. Two of these are described and illustrated, one of which is new in principle, being a differential form of the calorimeter. The accuracy of observation attained by this latter form is so considerable that it has been found possible to estimate directly the specific heats of the gases at constant volume to a close degree of accuracy.

An error incidental to the use of the method arising from the radiation of the substance, when surrounded by steam, to the walls of the calorimeter, is inquired into. It is shown that this affects the accuracy of the result to a very small degree, and is capable of easy estimation and elimination.

Further confirmation of the truth of the method is afforded in a comparison of experiments made in different forms of the steam calorimeter.

Various tables of constants are given to facilitate the use of the method, and the results of experiments on the density of saturated steam at atmospheric pressures, made directly in the calorimeter, are included. These are concordant with the deductions of Zeuner, based on Regnault's observations on the properties of steam, and were undertaken in the hope of affording reliable data on which to calculate the displacement effect on the apparent weight of the substance transferred from air to steam.

The communication is intended to provide a full account of the mode of application of the steam calorimeter.

VI. "On the Extension and Flexure of Cylindrical and Spherical Thin Elastic Shells." By A. B. BASSET, M.A., F.R.S. Received December 9, 1889.

(Abstract.)

The usual theory of thin elastic shells is based upon the hypothesis that the three stresses R, S, T, may be treated as zero, where R is the

normal traction perpendicular to the middle surface, and S and T are the two shearing stresses which tend to produce rotation about two lines of curvature of the middle surface. This hypothesis requires that these stresses should be at least of the order of the square of the thickness of the shell, for when this is the case they give rise to terms in the expression for the potential energy due to strain, which are proportional to the fifth power of the thickness, and which may be neglected, since it is usually unnecessary to retain powers of the thickness higher than the cube. It can be proved directly from the general equations of motion of an elastic solid, that this proposition is true in the case of a plane plate, provided the surfaces of the plate are not subjected to any pressures or tangential stresses, but there does not appear to be any simple method of establishing a similar proposition in the case of curved shells. I have therefore adopted this proposition as a fundamental hypothesis, and have endeavoured to establish its truth and to obtain a satisfactory theory of cylindrical and spherical shells in the following manner:—

Taking the case of a cylindrical shell, let OADB be a curvilinear rectangle described on the middle surface, of which the sides OA, BD are generators, and the sides AD, OB are circular sections. The resultant stresses per unit of length across the section AD consist of (1) a tension, T_1; (2) a tangential shearing stress, M_2; (3) a normal shearing stress, N_2; (4) a flexural couple, G_2, about AD; (5) a torsional couple H_1, perpendicular to AD; and the stresses across BD may be derived by interchanging the suffixes 1 and 2. Resolving along OA, OB, and the normal, and taking moments about these lines, we obtain the following equations,* viz.:—

$$\left.\begin{aligned}
\frac{dT_1}{dz}+\frac{1}{a}\frac{dM_1}{d\phi} &= X \\
\frac{1}{a}\frac{dT_2}{d\phi}+\frac{N_1}{a}+\frac{dM_2}{dz} &= Y \\
\frac{dN_2}{dz}+\frac{1}{a}\frac{dN_1}{d\phi}-\frac{T_2}{a} &= Z \\
\frac{1}{a}\frac{dG_1}{d\phi}+\frac{dH_1}{dz}+N_1 &= L \\
\frac{dG_2}{dz}+\frac{1}{a}\frac{dH_2}{d\phi}-N_2 &= M \\
(M_2-M_1)\,a-H_2 &= 0
\end{aligned}\right\}\ldots\ldots\ldots\ldots\ldots \text{(i)},$$

* Compare Besant, "On the Equilibrium of a Bent Lamina," 'Quart. Journ. Math.,' 1860.

where X,Y . . . denote certain expressions involving the bodily forces, such as gravity and the like, and the time variations of the displacements.

The values of the four couples may be calculated by a direct method, and are

$$\left.\begin{aligned} G_1 &= -\tfrac{4}{3}nh^3\mathfrak{F}, \quad G_2 = \tfrac{4}{3}nh^3(\mathfrak{E}+\mathfrak{A}/a) \\ H_1 &= -\tfrac{2}{3}nh^3(p+\varpi_3/a), \quad H_2 = \tfrac{2}{3}nh^3p \end{aligned}\right\} \cdots\cdots\cdots \text{(ii)}.$$

To explain the symbols involved in these equations, let u, v, w be the displacements along OA, OB, and the normal; σ_1, σ_2, ϖ_3 the extensional and shearing strains along and about these directions respectively; then putting $E = (m-n)/(m+n)$, the symbols in (ii) are defined by the following equations:—

$$\left.\begin{aligned} \mathfrak{A} &= \sigma_1+E\,(\sigma_1+\sigma_2), \quad \mathfrak{B} = \sigma_2+E\,(\sigma_1+\sigma_2) \\ \mathfrak{E} &= \lambda+E\,(\lambda+\mu), \qquad \mathfrak{F} = \mu+E\,(\lambda+\mu) \\ \lambda &= -\frac{d^2w}{dz^2}, \; \mu = -\frac{1}{a^2}\left(\frac{d^2w}{d\phi^2}+w\right)-\frac{E}{a}(\sigma_1+\sigma_2) \\ p &= -\frac{2}{a}\frac{d^2w}{dzd\phi}+\frac{1}{a}\frac{dv}{dz}-\frac{1}{a^2}\frac{du}{d\phi} \end{aligned}\right\} \cdots\cdots \text{(iii)}.$$

It is important to notice that the couples involve the extension of the middle surface as well as the change of curvature.

The expression for the potential energy is next found, and its value per unit of area of the middle surface is

$$\begin{aligned} W = {} & 2nh\{\sigma_1^2+\sigma_2^2+E\,(\sigma_1+\sigma_2)^2+\tfrac{1}{2}\varpi_3^2\} \\ & +\tfrac{2}{3}nh^3\{\lambda^2+\mu^2+E\,(\lambda+\mu)^2+\tfrac{1}{2}p^2\} \\ & +\tfrac{2}{3}nh^3(\mathfrak{A}\lambda'+\mathfrak{B}\mu'+\tfrac{1}{2}\varpi_3p') \\ & +\frac{4nh^3}{3a}(\mathfrak{A}\lambda+\mathfrak{B}\mu+\tfrac{1}{2}\varpi_3p) \cdots\cdots\cdots\cdots\cdots \text{(iv)}. \end{aligned}$$

The quantities λ', μ', p', depend partly upon quantities which define the bending, and partly upon the extension of the middle surface.

This expression is different from that obtained by Mr. Love, which arises from the fact that he has omitted to take into account several terms depending upon the product of the extensions and h^3. It will be noticed that (iv) reduces to the second line when the middle surface is inextensible, and in this case agrees with the expression obtained by Lord Rayleigh.*

* 'Roy. Soc. Proc.,' Dec., 1888.

The variational equation of motion may be written

$$\delta W + \delta \mathfrak{T} = \delta U + \delta \mathfrak{W} \ldots\ldots\ldots\ldots\ldots\ldots \text{(v)}$$

where $\delta \mathfrak{T}$ is the term depending on the time variations of the displacements, δU is the work done by the bodily forces, and $\delta \mathfrak{W}$ is the work done upon the edges of the portion of the shell considered, by the stresses arising from the action of contiguous portions of the shell.

Applying (v) to a curvilinear rectangle bounded by four lines of curvature, and working out the variation in the usual way, the line integral part will determine the values of the edge stresses $T_1, T_2 \ldots$, in terms of the displacements, and ought also to reproduce the values of the couples which we have already obtained; and the surface integral part will give the three equations of motion in terms of the displacements. These results furnish a test of the correctness of the work, and also of the fundamental hypothesis upon which the theory is based; for if we substitute the values of the edge stresses in terms of the displacements in the first three of (i), we ought to reproduce the equations of motion which we have obtained by means of the variational equation; and this is found to be the case.

The boundary conditions can be obtained by Stokes' theorem, which enables us to prove that it is possible to apply a certain distribution of stress to the edge of a thin shell, without producing any alteration in the potential energy due to strain.

The general equations, owing to their exceedingly complicated character, do not, except in special cases, readily lend themselves to the solution of mathematical problems; but, for the purpose of throwing some light upon the question raised by Mr. Love, as to the impossibility of satisfying the boundary conditions at a free edge, when a curved shell is vibrating in such a manner that its middle surface experiences no extension nor contraction throughout the motion, I have considered the following statical problem:—

A heavy cylindrical shell, whose cross section is a semi-circle, is suspended by vertical bands attached to its straight edges, so that its axis is horizontal, and is deformed by its own weight; required the strain produced.

We shall assume that the displacement at every point of the middle surface lies in a plane perpendicular to the axis, and we shall suppose that the necessary stresses are applied to the circular edges.

Measuring ϕ from the lowest point and putting R for the change of curvature along a circular section, we find that

$$\frac{R}{\sigma_2} = \frac{E}{a} + \frac{3a(\frac{1}{2}\pi - \phi\sin\phi - \cos\phi)}{h^2\{\frac{1}{2}\pi - \cos\phi + \frac{3}{2}E(\frac{1}{2}\pi - \phi\sin\phi - \cos\phi)\}} \ldots \text{(vi)}.$$

Since h is small compared with a, this equation shows that the

change of curvature is large in comparison with the extension, except at points in the neighbourhood of the edge, where $a\ (\frac{1}{2}\pi-\phi)$ is comparable with h.

It is also shown that the tension T_1 parallel to the axis, and the couple G_2 about a circular section do not vanish at the circular edges, but have finite values; and therefore a tension and a couple of the proper amount, which tends to produce synclastic curvature of the generating lines must be applied at the circular edges. If, therefore, this force and couple were removed, anticlastic curvature of the generating lines would be produced, and this would involve extension of the middle surface parallel to the axis. It is, however, obvious that a thin shell, under these circumstances, does not assume a saddle-back form, and therefore the anticlastic curvature, and the extension upon which it depends, must be exceedingly small, except in the neighbourhood of the circular edges.

The difficulty of satisfying the boundary conditions at a curved free edge, when the middle surface is supposed to be inextensible, partly arises from the fact that it is impossible for the flexural couple about the curved edge to vanish, unless some extension or contraction takes place in the neighbourhood of the edge; but the inference to be drawn from the statical problem considered above is, that when a thin shell, whose edges are free, is vibrating, the amplitudes of those terms upon which the extension depends are small in comparison with the amplitudes of those terms upon which the bending depends. Moreover, a variety of results which have been obtained during recent years indicate, that the pitch of notes which depend upon extension is very high, compared with the pitch of notes which depend upon flexure; and this circumstance, combined with the smallness of the amplitudes of the extensional vibrations, points to the conclusion that the former notes are usually feeble in comparison with the latter.

The values of the edge stresses and the equations of motion are also obtained for a spherical shell, but the work is the same as in the case of a cylindrical shell, except in matters of detail.

The Society adjourned over the Christmas Recess to Thursday, January 9th, 1890.

Presents, December 19, 1889.

Transactions.

Halle:—Verein für Erdkunde. Mitteilungen. 1889. 8vo. *Halle* 1889. The Verein.

Helsingfors:—Finska Vetenskaps-Societet. Acta. Tomus XVI. 4to. *Helsingforsiæ* 1888; Öfversigt, 1887-88. 8vo. *Helsingfors* 1888. The Society.